

A Simple Proof of the Beal Conjecture.
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The Beal Conjecture can be stated as follows: For positive integers a, b, c, x, y and z , if $a^x + b^y = c^z$, and a, b and c are co-prime, then x, y and z are not all greater than 2.

Proof:

Let all variables represent positive integers and let unity be defined as: $1 = (T/T)$, where $T \neq 1$. Then, assuming both factorability and coprimality, we have:

$$\begin{aligned}
 a^x + b^y &= c^z = \\
 ((T/T)a^{(x/2)})^2 + ((T/T)b^{(y/2)})^2 &= ((T/T)c^{(z/2)})^2 = \\
 (T(c/T)^{((z/2)\ln(c)/(\ln(T)) - 1)/(\ln(c)/(\ln(T)) - 1)})^2 &. \tag{1}
 \end{aligned}$$

Eliminating the outermost parenthesis in (1) may or may not involve the cancelled variable T , and therefore results in either:

$$\begin{aligned}
 (T^2/T^2)a^x + (T^2/T^2)b^y &= (T^2/T^2)c^z = \\
 T^2(c/T)^{((z\ln(c)/(\ln(T)) - 2)/(\ln(c)/(\ln(T)) - 1))} &, \tag{2}
 \end{aligned}$$

or

$$\begin{aligned}
 (T/T)a^x + (T/T)b^y &= (T/T)c^z = \\
 T(c/T)^{((z\ln(c)/(\ln(T)) - 1)/(\ln(c)/(\ln(T)) - 1))} &. \tag{3}
 \end{aligned}$$

Here we note that $1 = (T/T) = (c/c)$, and that $T = c$ must therefore be allowable. We also note that the logarithms preventing $T = c$ cease to exist if and only if $z = 2$ in (2), and $z = 1$ in (3), which gives us both:

$$(T^2/T^2)a^x + (T^2/T^2)b^y = (T^2/T^2)c^2 = T^2(c/T)^2, \tag{4}$$

and

$$(T/T)a^x + (T/T)b^y = (T/T)c = T(c/T). \tag{5}$$

$T = c$ is now clearly allowable, and simplifying (4) and (5) shows that we are left with either:

$$a^x + b^y = c^2, \tag{6}$$

or

$$a^x + b^y = c, \tag{7}$$

which proves the conjecture.

Notes:

The term involving logarithms in (1) can be derived as follows:

$$\left(\frac{T}{T}c^{\frac{z}{2}}\right)^2 =$$

$$\left(T\left(\frac{c}{T}\right)^{\frac{\ln(c^{\frac{z}{2}}/T)}{\ln(c/T)}}\right)^2 =$$

$$\left(T\left(\frac{c}{T}\right)^{\frac{\ln(c^{\frac{z}{2}}/T)/\ln(T)}{\ln(c/T)/\ln(T)}}\right)^2 =$$

$$\left(T\left(\frac{c}{T}\right)^{\left(\frac{\ln(c^{\frac{z}{2}})/\ln(T)}{\ln(T)} - \frac{\ln(T)/\ln(T)}{\ln(c)/\ln(T)} - \frac{\ln(T)/\ln(T)}{\ln(T)}\right)}\right)^2 =$$

$$\left(T\left(\frac{c}{T}\right)^{\left(\left(\frac{z}{2}\ln(c)/\ln(T)\right) - 1\right)/\left(\ln(c)/\ln(T) - 1\right)}\right)^2.$$

The last three terms in this derivation are called “cohesive terms” because unlike the first two terms, their form prevents cancelled variables from being “crossed out” and “lost”. Thus, they alone can represent factorability and coprimality in a manner that is both effective, and logically consistent. They are also unique in that they preclude ineffectual operations such as multiplication and/or division by unity, and are in fact, the only algebraic terms whose variables are well defined by their own unique domains.