A Special Polygonal Number Counting Function Involving
the Fine Structure Constant and the Proton to Electron Mass Ratio.

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Abstract:

Polygonal numbers are among the most studied numbers in history, so it is important to ask how many of rank greater than 2 there are under a given number x. In this paper, we introduce the function designated as $\mathcal{B}(x)*\left(1-\frac{\alpha}{\mu-2*e}\right)$, which not only answers this question, but relates it to both the ***fine structure constant*** α and the ***proton to electron mass ratio*** μ.

Article:

Let polygonal numbers of rank greater than 2 be defined as the various different numbers:

$\wp_{n>2}^{r>2}=6,9,10,12,15,16,18,21,22,24,25,27,28,30$ ... which are generated by the formula:

$\left(\frac{n}{2}-1\right)*r^2-\left(\frac{n}{2}-2\right)*r$, when integers n and r are greater than 2 ,

and let $\varpi(x)$ represent how many such numbers there are less than or equal to a given number x .

Then, $\varpi(x)\sim\mathcal{B}(x)=x-(\alpha*\pi*e+e)^{-1}*x-\frac{1}{2}*\sqrt{x-(\alpha*\pi*e+e)^{-1}*x}$

where $\alpha\approx137.03599908451^{-1}$ is the ***fine structure constant***.

The table below represents $\varpi(x)$ approximated by $\mathcal{B}(x)$:

x	$\varpi(x)$	$\mathcal{B}(x)$	Difference	% of error
10	3	5	2	.666666667
100	57	60	3	.052631579
1,000	622	628	6	.009646302
10,000	6,357	6, 364	7	.001101148
100,000	63,889	63,910	21	.000328695
1,000,000	639,946	639,965	19	.000029690
10,000,000	6,402,325	6,402,388	63	.000009840
100,000,000	64,032,121	64,032,528	407	.000006356
1,000,000,000	640,349,979	640,352,643	2,664	.000004160
10,000,000,000	6,403,587,409	6,403,612,945	25,536	.000003988
100,000,000,000	64,036,148,166	64,036,403,036	254,870	.000003980
1,000,000,000,000	640,362,343,980	640,364,895,519	2551539	.000003984

Then, since the percentage of error is approaching the constant: $\frac{\alpha}{\mu - 2*e} = .0000039860645427873$ where $\mu \approx 1836.152672471880$ is the ***proton to electron mass ratio***, we also have the function:

$$\varpi(x) \sim \mathcal{B}(x) * \left(1 - \frac{\alpha}{\mu - 2*e}\right) =$$

$$\left(x - (\alpha * \pi * e + e)^{-1} * x - \frac{1}{2} * \sqrt{x - (\alpha * \pi * e + e)^{-1} * x}\right) * \left(1 - \frac{\alpha}{\mu - 2*e}\right),$$

which results in the table:

x	$\varpi(x)$	$\mathcal{B}(x) * \left(1 - \frac{\alpha}{\mu - 2*e}\right)$	Difference	% of error
10	3	5	2	.666666667
100	57	60	3	.052631579
1,000	622	628	6	.009646302
10,000	6,357	6, 364	7	.001101148
100,000	63,889	63,910	21	.00032869508
1,000,000	639,946	639,963	17	.00002656474
10,000,000	6,402,325	6,402,362	37	.00000577915
100,000,000	64,032,121	64,032,273	152	.00000237381
1,000,000,000	640,349,979	640,350,090	111	.00000017334
10,000,000,000	6,403,587,409	6,403,587,420	11	.00000000172
100,000,000,000	64,036,148,166	64,036,147,783	−383	−.00000000598
1,000,000,000,000	640,362,343,980	640,362,342,983	−997	−.00000000156

Note the *very small* and rapidly decreasing percentage of error. Clearly, this is extraordinary and compelling evidence that both the fine structure constant and the proton to electron mass ratio are not only "physical constants", but *mathematical* constants as well. ■

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