Proof of Beal's Conjecture.

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Abstract:

We demonstrate that all three terms in "Beal's Conjecture" are implicitly squares under second degree radicals. We then define the "trivial" common factor unity, apply that definition to an arbitrarily chosen term, and enforce that definition by introducing a newly discovered logarithmic identity whose properties demonstrate the truth of the conjecture.

Description:

Beal's Conjecture can be stated as follows: For positive integers: $a, b, c, x, y, z,$
if: $a^x + b^y = c^z$ and: a, b, c are co-prime, then: x, y, z are not *all* greater than 2.

Proof:

Let all variables herein represent positive integers such that: $a^x < b^y < c^z$.
Then, the "Beal equations":

$$a^x + b^y = \sqrt{((b^y - a^x)^2 + 4a^x b^y)} = \sqrt{(c^z)^2}\ , \tag{1}$$

$$c^z - a^x = \sqrt{((a^x + c^z)^2 - 4a^x c^z)} = \sqrt{(b^y)^2} \tag{2}$$

and:

$$c^z - b^y = \sqrt{((b^y + c^z)^2 - 4b^y c^z)} = \sqrt{(a^x)^2}\ , \tag{3}$$

demonstrate that *any* sum or difference of two terms expressed as a single term is implicitly a square under a second degree radical.

Here, it will be assumed that: a, b, c are co-prime, so that the only common factor possible is the "trivial" unity, which cannot be defined in terms of itself, and must therefore be defined as:

$$1 = \frac{T}{T}\ \text{, where: } T > 1. \tag{4}$$

Arbitrarily choosing equation (1) to serve as an example, we now restate the far right hand side of that equation so that the "trivial" common factor: $1 = \frac{T}{T}$ and its newly discovered logarithmic consequences are actually *represented*. This gives us:

$$\sqrt{(c^z)^2} = c^z = \left(\frac{T}{T}\right) c^z = T\left(\frac{c}{T}\right)^{\frac{\frac{(z)\ln(c)}{\ln(T)}-1}{\frac{\ln(c)}{\ln(T)}-1}} , \tag{5}$$

where z *must* represent odd numbers only, since all even values of z would imply and therefore require that the terms consist of two equal parts being multiplied together, as in the *only* possible alternate interpretation:

$$\sqrt{(c^z)^2} = \left(c^{\frac{z}{2}}\right)^2 = \left(\left(\frac{T}{T}\right) c^{\frac{z}{2}}\right)^2 = \left(T\left(\frac{c}{T}\right)^{\frac{\frac{\left(\frac{z}{2}\right)\ln(c)}{\ln(T)}-1}{\frac{\ln(c)}{\ln(T)}-1}}\right)^2 , \tag{6}$$

where z *must* represent even numbers only in order to maintain the integrality of the expression $c^{\frac{z}{2}}$. Thus, equations (5) and (6) are different, distinct and separate cases, which cannot be considered or satisfied simultaneously, since they are two *mutually exclusive possibilities*.

Here, we note that the expressions involving logarithms "cancel out" if and only if: $z = 1$ in equation (5) and: $z = 2$ in equation (6), which results in both:

$$\sqrt{(c^1)^2} = c = \left(\frac{T}{T}\right) c = T\left(\frac{c}{T}\right) \tag{7}$$

and:

$$\sqrt{(c^2)^2} = (c)^2 = \left(\left(\frac{T}{T}\right) c\right)^2 = \left(T\left(\frac{c}{T}\right)\right)^2 , \tag{8}$$

and since equations (7) and (8) *are* equations (5) and (6), properly evaluated at: $z = 1$ and: $z = 2$ respectively, we can unequivocally state that:

equation (7) is the case where: $z = \{1\}$,
equation (8) is the case where: $z = \{2\}$,
equation (5) is the case where: $z = \{3,5,7,9 \ldots\}$ and
equation (6) is the case where: $z = \{4,6,8,10 \ldots\}$,

and that the above four cases clearly include all possible values of z in "Beal's Conjecture".

Now, in *all* cases both logical and mathematical, it must be possible to substitute: 1 for: 1 .
However, substituting: $\frac{c}{c}$ for: $\frac{T}{T}$ is possible *only* in equations (7) and (8), which results in both:

$$\sqrt{(c^1)^2} = c = \left(\frac{c}{c}\right)c = c\left(\frac{c}{c}\right) \tag{9}$$

and:

$$\sqrt{(c^2)^2} = (c)^2 = \left(\left(\frac{c}{c}\right)c\right)^2 = \left(c\left(\frac{c}{c}\right)\right)^2. \tag{10}$$

Thus, it is clear that substituting: $\frac{c}{c}$ for: $\frac{T}{T}$ is possible if and only if: $z = 1$ or: $z = 2$.
Moreover, the *impossibility* of substituting: $\frac{c}{c}$ for: $\frac{T}{T}$ in equations (5) and (6) implies that: $1 \neq 1$, which is a *contradiction*, and since this contradiction occurs if and only if the assumption is made that: x, y, z are all greater than 2 when: $a^x + b^y = c^z$ and: a, b, c are co-prime, that assumption must be wrong and the conjecture has been proved. ■

Note:

The terms involving logarithms in equations (5) and (6) can be similarly derived as follows:

$$\left(\frac{T}{T}\right)c^z = T\left(\frac{c}{T}\right)^{\frac{\ln\left(\frac{c^z}{T}\right)}{\ln\left(\frac{c}{T}\right)}} = T\left(\frac{c}{T}\right)^{\frac{\frac{\ln\left(\frac{c^z}{T}\right)}{\ln(T)}}{\frac{\ln\left(\frac{c}{T}\right)}{\ln(T)}}} = T\left(\frac{c}{T}\right)^{\frac{\frac{\ln(c^z)}{\ln(T)} - \frac{\ln(T)}{\ln(T)}}{\frac{\ln(c)}{\ln(T)} - \frac{\ln(T)}{\ln(T)}}} = T\left(\frac{c}{T}\right)^{\frac{\frac{(z)\ln(c)}{\ln(T)} - 1}{\frac{\ln(c)}{\ln(T)} - 1}} .$$

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